

Non-Gaussianities in extended D-term inflation

Francis Bernardeau and Tristan Brunier

Service de Physique Théorique, CEA/DSM/SPhT,

Unité de recherche associée au CNRS, CEA/Saclay 91191 Gif-sur-Yvette cedex

We explore extensions of hybrid inflationary models in the context of supersymmetric D-term inflation. We point out that a large variety of inflationary scenarios can be encountered when the field content is extended. It is not only possible to get curvaton type models but also scenarios in which different fields, with nontrivial statistical properties, contribute to the primordial curvature fluctuations. We explore more particularly the parameter space of these multiple field inflationary models. It is shown that there exists a large domain in which significant primordial non-Gaussianities can be produced while preserving a scale free power spectrum for the metric fluctuations. In particular we explicitly compute the expected bi- and trispectrum for such models and compared the results to the current and expected observational constraints. It is shown that it is necessary to use both the bi- and tri-spectra of CMB anisotropies to efficiently reduce their parameter space.

PACS numbers: 98.80.-k, 98.80.Cq, 98.80.Es, 98.80.Hw

I. INTRODUCTION

The new generations of high precision Cosmic Microwave Background (CMB) measurements opens a new window to the physics of the early universe and more specifically to the physics of inflation. Up to now, the shape of the power spectrum has been the prime object of interest in order to confront theory and observation. Any observed departure from a scale free, Harrison-Zel'dovich type, power spectrum would indeed give a precious hint on the shape of the inflaton potential and subsequently on models of inflation [36]. But needless is to say that detection of primordial non-Gaussianities in the CMB temperature anisotropies or polarization would also carry precious indications regarding the inflaton sector.

It has been shown however that in its simplest formulation inflation does not lead to significant non-gaussianities (NG). This result has been rigorously established at the level of the bispectrum over the last few years [1, 2, 3] and recently it has been extended to trispectrum [4, 5]. To be more precise the low level of primordial NG that standard inflation is expected to induce is unlikely to be detected in practice due to the subsequent evolution of the metric whether it is at superhorizon scale, during or after the inflationary phase or during the complex stages of the recombination [6, 7].

It appears then useful to have at our disposal sets of models in which significant primordial non-Gaussianities can be produced, that is an amount of NG that clearly exceeds what the nonlinear gravitational evolution is inducing. This appears to be the case for instance for models built out of actions close to the Dirac-Born-Infeld one [8, 9]. Those models however rely on non trivial extension of the kinetic part of the Lagrangian. Although that would be a ground breaking discovery to see such effects, it remains that the appearance of primordial NG does not require the use of such non-standard extension of the low-energy field equation. The curvaton model offers another path to possible large amount of primordial NG [10]. Here primordial non-Gaussianities are induced

lately, at the time the energy density of the curvaton field overcome the inflaton decay products. The efficiency of such a mechanism therefore depends on sectors of the theory that are not directly relevant to the inflation.

A third possibility has recently been advocated which is entirely determined in the inflationary sector. It is provided by some models of multiple field inflations. The mechanism at play has been described in various papers [11, 12]. The difficulty so far is to identify more precisely models in which such a scenario could take place. In [13] explicit Lagrangian (with canonical kinetic terms) were proposed for which significant non-Gaussianities can be generated during the inflationary phase. Although it was demonstrated that it was possible to get them during the whole inflationary stage, the most efficient (and in some sense most natural) model in which such a possibility was achieved was based on an hybrid inflation type construction. In this case significant non-Gaussianities can be transferred to the metric fluctuations at the very end of inflation, at the time of the tachyonic instability [37]. The model proposed in [13] was however purely phenomenological and was not motivated by high energy physics constructions.

We propose here to explore a bit further high-energy physics motivations for multiple-field hybrid inflation. Hybrid inflationary models introduced by A. Linde [14] provided a way to circumvent one fundamental issue in standard inflation with simple potentials, that is that the Vacuum Expectation Values (VEVs) of the inflaton field is bound to be of the order of the Planck mass during the inflationary period [15]. This is the case in particular with quadratic or quartic potentials. With hybrid inflation the end of inflation is triggered by a tachyonic instability and the VEVs of the fields can be made as small as one wishes (at the expense of a power spectrum index very close to unity). Hybrid inflation has received some further theoretical justification because it naturally appears in various high energy setups. In the context of global supersymmetry, it is indeed possible to construct potentials which allow a viable inflationary period, either

based on the F- or the D-term of the potentials [16, 17]. The aim of this paper is to take advantage of such theoretical frameworks to build models of multiple field inflation and explore their phenomenological consequences.

The plan of the paper is the following. We first succinctly review the construction and constraints associated to supersymmetric hybrid models. We then present possible extensions of these models and their phenomenological consequences. The amount of non-Gaussianity they can lead to is precisely computed and confronted to observational constraints.

II. SUPERSYMMETRIC HYBRID INFLATION

In a supersymmetric context, two scenarios have been identified that lead to inflationary models that reproduce hybrid type potentials. Those scenarios are the F- and D- term inflationary models [18]. They allow in particular to have inflation while the field VEVs remain small compared to the Planck mass. In such a case the expression of the low energy Lagrangian is safe from corrective terms originating from supergravity effects (e.g. through the Kähler potential) or high-order curvature terms. This is the domain we will stick to throughout this paper.

To be more specific, the F- and D-term inflation are based on the specific global supersymmetry breaking mechanisms. Once symmetry breaking occurs, the potential has to be corrected for one-loop radiative correction terms. These corrections are encapsulated in the formula of Coleman-Weinberg derived in [19]. For soft supersymmetry breaking schemes (where the trace of the mass matrices remain unchanged) one is left with logarithmic corrections to the potential. This is precisely what we have for F- and D-term inflationary models.

The F-term inflation is based on a superpotential of the form [16],

$$W = \lambda \mathcal{S} \bar{\phi} \phi - \mu^2 \mathcal{S} \quad (1)$$

where \mathcal{S} , $\bar{\phi}$ and ϕ correspond to the scalar degrees of freedom. The resulting potential is the following. It comes from F-term contribution only,

$$V = \lambda^2 |\mathcal{S}|^2 (|\bar{\phi}|^2 + |\phi|^2) + |\lambda \bar{\phi} \phi - \mu^2|^2. \quad (2)$$

The energetic landscape it leads to shows that there exists a flat direction along which the potential is constant (at tree order) but non-vanishing. It is for a such field configuration that the inflationary phase can take place. This flat direction is obtained for $\phi = \bar{\phi} = 0$. It corresponds to a stable minimum (e.g. all masses are positive) as long as

$$\lambda^2 |\mathcal{S}|^2 > \mu^2 \lambda, \quad (3)$$

that is $|\mathcal{S}| > \mathcal{S}_c$ with $\mathcal{S}_c = \mu/\sqrt{\lambda}$. Such a vacuum state obviously breaks global supersymmetry. It implies that

corrective terms due to non-vanishing radiative corrections have to be added to the potential. At one-loop order, they approximatively read,

$$V_{1\text{-loop}} \approx \frac{\lambda^4 \mathcal{S}_c^4}{16\pi^2} \log \frac{|\mathcal{S}|}{\mathcal{S}_c}. \quad (4)$$

This is thanks to this extra contribution that such a setting can lead to a viable inflationary model. It induces a (slow) roll of the field \mathcal{S} towards the origin. In this scenario inflation terminates [38] when $|\mathcal{S}| = \mathcal{S}_c$ due to the tachyonic instability that then appears. During the inflationary phase the potential can be approximated by,

$$V_{\text{F-term}}(\mathcal{S}) = \lambda^2 \mathcal{S}_c^4 \left(1 + \frac{\lambda^2}{16\pi^2} \log \frac{|\mathcal{S}|}{\mathcal{S}_c} \right), \quad (5)$$

to a very good approximation.

The mechanism that leads to a D-term inflation is very similar [17]. This model is based on the introduction of a $U(1)$ symmetry group and a non-vanishing Fayet-Iliopoulos term leading to a non-zero D-term in the potential. Another family of inflationary models can then be constructed with the help of three fields, one, \mathcal{S} , with zero charge, a field ϕ with a positive charge and a field $\bar{\phi}$ with a negative one under the $U(1)$ symmetry and with the following superpotential,

$$W = \lambda \mathcal{S} \bar{\phi} \phi. \quad (6)$$

It leads to the potential,

$$\begin{aligned} V &= V_F + V_D \\ &= \lambda^2 |\mathcal{S}|^2 (|\bar{\phi}|^2 + |\phi|^2) + \lambda^2 |\bar{\phi} \phi|^2 + \frac{g^2}{2} (|\phi|^2 - |\bar{\phi}|^2 + \xi)^2 \end{aligned}$$

In a way similar to the F-term inflation, the potential exhibits a flat stable direction when,

$$\phi = 0, \quad \bar{\phi} = 0, \quad |\mathcal{S}| > \mathcal{S}_c \quad (8)$$

with

$$\mathcal{S}_c = \frac{g}{\lambda} \sqrt{\xi}. \quad (9)$$

In this case the inflaton potential is given by,

$$V_{\text{D-term}}(\mathcal{S}) = \frac{\lambda^4 \mathcal{S}_c^4}{2g^2} \left(1 + \frac{g^2}{8\pi^2} \log \frac{|\mathcal{S}|}{\mathcal{S}_c} \right). \quad (10)$$

This is the context in which we will work in the following.

A. Conditions to have a valid D-term inflation

A number of conditions should be met for such models to provide us with valid inflationary models: the number of e-folds it leads to should be large enough and the amplitude of the metric fluctuations is constrained. These are obviously constraints that are generic for all inflationary models. There are however two difficulties that

one wants to avoid in the context of D-term inflation. Namely one does not want the VEVs of the fields to exceed the Planck scale so that global supersymmetry remains a valid description of the Lagrangian[39] and the formation of too massive cosmic strings at the end of the inflationary phase should also be avoided.

The first requirement leads us to assume that not only \mathcal{S}_c is small but also that the first term of the inflaton potential, the constant one, is dominating over the second contribution. This is possible if

$$g^2 \ll 1. \quad (11)$$

With such a constraint the end of inflation takes place when $|\mathcal{S}| = \mathcal{S}_c$. The number of e-folds between horizon crossing for the modes of interest (the ones that are responsible for the observed large-scale structure) and the end of inflation can then be easily computed. It is given by

$$N_e = \frac{8\pi^2}{g^2 M_{\text{Pl}}^2} (\mathcal{S}_*^2 - \mathcal{S}_c^2), \quad (12)$$

where \mathcal{S}_* is the modulus of \mathcal{S} during horizon crossings and M_{Pl} is the Planck mass, $M_{\text{Pl}}^2 = 1/(8\pi G)$. The amplitude of the metric fluctuations \mathcal{R}_* for the modes of interest is observed to be of the order of 10^{-4} (see next section for more details). It implies that

$$\frac{4\pi^2}{\sqrt{3}} \frac{\xi \mathcal{S}_*}{g M_{\text{Pl}}^3} \approx 10^{-4}. \quad (13)$$

These two relations lead to the constraint,

$$\frac{16\pi^4}{3} \frac{\xi^2}{M_{\text{Pl}}^4} \left[\frac{N_e}{8\pi^2} + \frac{\xi}{\lambda^2 M_{\text{Pl}}^2} \right] \approx 10^{-8}. \quad (14)$$

This expression exhibits two regimes. One in which $\xi/(\lambda^2 M_{\text{Pl}}^2) \ll N_e/(8\pi^2)$ and then the energy scale of ξ is fixed, of the order of $10^{-5} M_{\text{Pl}}^2$, and one in which $\xi/(\lambda^2 M_{\text{Pl}}^2) \gg N_e$ and where $\xi \sim 0.5 \lambda^{2/3} 10^{-3} M_{\text{Pl}}^2$. But ξ is precisely the energy scale of the strings that form at the end of inflation. The constraint on the string content of the Universe favors the second case (see [20, 21] for details. It is to be noted also that it might be possible to circumvent this constraint).

It is to be noted that in this latter case \mathcal{S}_* is nearly equal to \mathcal{S}_c . More precisely we have,

$$\mathcal{S}_* \approx \mathcal{S}_c \left(1 + \frac{N_e g^2 M_{\text{Pl}}^2}{16\pi^2 \mathcal{S}_c^2} \right). \quad (15)$$

We will see that this is of importance for the phenomenological consequences of D-term extended models.

III. EXTENSION OF THE FIELD CONTENT

Here we now propose to extend the field content involved in the inflationary sector of the theory. Following

a conservative approach regarding the high energy sector of the theory we allow ourselves to introduce up to cubic terms only in the superpotential. For instance a superpotential of the form,

$$W = \lambda_1 \mathcal{S}_1 \bar{\phi} \phi + \mu_2^2 \mathcal{S}_2, \quad (16)$$

is a priori legitimate. It involves two light fields that coexist during the inflationary phase. If μ_2 is smaller than the Hubble value during the inflationary phase then super-Hubble fluctuations can be generated in the \mathcal{S}_2 field. Whether those fluctuations will be observable or not depends on the subsequent evolution of the \mathcal{S}_2 field. In particular, because \mathcal{S}_2 is a massive field, its energy density can eventually dominate over the inflaton decay products if those are relativistic particles. The \mathcal{S}_2 field can then imprint its fluctuations on the metric fluctuations before it finally decays. This is the curvaton mechanism. This possibility, and its phenomenological consequences, have been extensively described in the literature [10]. They depend largely on the coupling of the extra fields to the other fields of the theory. Such a theory is then not entirely predictive from the sole knowledge of the inflationary sector of the theory.

A. Multiple field models

The expression (16) is however not the only possible extension of (6). For instance nothing prevents the introduction of multiple light fields that are coupled together to the same charged $U(1)$ fields,

$$W = \sum_i \frac{\nu_i}{3} \mathcal{S}_i^3 + \lambda \left(\sum_i \alpha_i \mathcal{S}_i \right) \bar{\phi} \phi. \quad (17)$$

Obviously if ν_i is small enough, the corresponding \mathcal{S}_i field can participate in the inflaton (depending on its initial VEV). The corresponding upper bound for ν_i for such a possibility to occur is defined in such a way that, when, say, the VEV of \mathcal{S}_i is below the Planck scale, the contribution of the quartic potential it induces is negligible against the radiative correction term. It leads to the constraint,

$$\nu_i^2 \ll \lambda^4. \quad (18)$$

The fields for which ν_i is above this bound will rapidly roll towards the origin but they still can develop significant super-Hubble fluctuations as long as ν_i is smaller than unity. This would not be the case otherwise.

It is then possible to distinguish three sets of fields, those with a very small coupling constant ν ; they can potentially be part of the inflaton field; the ones with intermediate values; they will not contribute to the inflaton but still develop significant super-Hubble fluctuations and finally those with a large coupling constant will not develop any significant fluctuations and will not

play any role. In the following we will be interested in the second set of fields.

To simplify the presentation let us assume that there

is one field \mathcal{S}_1 with a vanishing coupling constant ν_1 and one field \mathcal{S}_2 with a large - but still smaller than unity - ν_2 .

Then the potential takes the form,

$$V = V_{1\text{-loop}} + \lambda^2 |\cos \theta \mathcal{S}_1 + \sin \theta \mathcal{S}_2|^2 (|\bar{\phi}|^2 + |\phi|^2) + \lambda^2 \cos^2 \theta |\phi|^2 |\bar{\phi}|^2 + |\nu_2 \mathcal{S}_2^2 + \lambda \sin \theta \phi \bar{\phi}|^2 + \frac{g^2}{2} (|\phi|^2 - |\bar{\phi}|^2 + \xi)^2 \quad (19)$$

where θ is the mixing angle of \mathcal{S}_1 and \mathcal{S}_2 encoded in the α_i parameters.

As for the previous case, ϕ is a very massive field whose VEV is driven to 0. The previous expression then simplifies and we are left with the effective potential,

$$V = V_{1\text{-loop}} + \nu_2^2 |\mathcal{S}_2|^4 + \lambda^2 |\cos \theta \mathcal{S}_1 + \sin \theta \mathcal{S}_2|^2 |\bar{\phi}|^2 + \frac{g^2}{2} (-|\bar{\phi}|^2 + \xi)^2 \quad (20)$$

involving the (complex scalar) fields \mathcal{S}_1 , \mathcal{S}_2 and $\bar{\phi}$. We see that it involves a new term, the self coupling term of the \mathcal{S}_2 field. It drives the VEV of this field to 0 during the inflationary period. The expression of $V_{1\text{-loop}}$ is then left unchanged compared to the single inflationary field case. As for the curvaton model, the \mathcal{S}_2 direction behaves like an isocurvature direction during the inflationary period. Because of the mixing term in the \mathcal{S}_i - $\bar{\phi}$ coupling though, the \mathcal{S}_2 fluctuations can lead to visible

effects irrespectively of the subsequent evolution of this field. To be more precise, the inflation terminates when $\cos \theta \mathcal{S}_1 + \sin \theta \mathcal{S}_2$ reaches the critical value \mathcal{S}_c . In the context we are considering, the inflationary period then ends almost instantaneously because of the tachyonic instability that subsequently develops [22, 23]. The end time of inflation is then modulated by both fluctuations in the \mathcal{S}_1 and \mathcal{S}_2 directions. This is the mechanism with which initial isocurvature fluctuations are transferred into the adiabatic modes.

The metric fluctuations that are then induced are a combination of \mathcal{S}_1 and \mathcal{S}_2 fluctuations. As long as the only leading order metric effects are taken into account, the formal expression of the metric fluctuations can be easily computed using the δN formalism [24, 25, 26]. The induced metric fluctuations then read

$$\delta N(\delta \mathcal{S}_1, \delta \mathcal{S}_2) = \int_{\text{inflation traj.}} H(\mathcal{S}_1^{(0)} + \delta \mathcal{S}_1, \delta \mathcal{S}_2) dt - \int_{\text{inflation traj.}} H(\mathcal{S}_1^{(0)}, 0) dt, \quad (21)$$

where $\mathcal{S}_1^{(0)}$ is the zero mode trajectory of the field. In the context we are interested in the expression of δN can easily be explicitated at leading order in the field fluctuations. It is given by,

$$\delta N = - \left. \frac{3H^2}{V_{,\varphi}} \right|_{\text{Horizon crossing}} \delta \varphi + \left. \frac{3H^2}{V_{,\varphi}} \right|_{\text{end of inflation}} \tan \theta \delta \chi_1. \quad (22)$$

where φ and χ are the (canonically defined) fluctuations of the fields \mathcal{S}_1 and \mathcal{S}_2 in the direction (in the complex plane) of $\mathcal{S}^{(0)}$. This direction, without loss of generality, can be defined as the real axis,

$$\varphi = \sqrt{2} \text{Re}(\mathcal{S}_1), \quad (23)$$

and the real part of \mathcal{S}_2 ,

$$\chi_1 = \sqrt{2} \text{Re}(\mathcal{S}_2), \text{ and } \chi_2 = \sqrt{2} \text{Im}(\mathcal{S}_2). \quad (24)$$

The imaginary parts of those fields are genuine degree of freedom that will develop super-Hubble correlations

as well. They will affect though the expression of δN at quadratic order only, as any other couplings to the metric would. At the level of our description the imaginary part of φ will not play any role. The one of χ will do however, because it affects the non-Gaussian properties of χ_1 as we will discover.

What is important here to realize is that in the parameter domain favored by the constraints on the cosmic strings contribution to the CMB anisotropies as discussed previously, the two coefficients, $3H^2/V_{,\varphi}|_{\text{horizon crossing}}$ and $3H^2/V_{,\varphi}|_{\text{end of inflation}}$

are, almost, equal. Moreover the two fields are independent of one another; their fluctuations have roughly the same spectrum, to the difference in their mass, e.g. η parameter. The generated metric fluctuations are then the superposition of these two contributions the relative weight is driven by a free parameter, the mixing angle between the two fields, θ . The amplitude of the induced metric fluctuations per unit log scale in k , \mathcal{P}_0 , is then typically given by [40],

$$\mathcal{P}_0 \sim (1 + \tan^2 \theta)^{1/2} \frac{3H^3}{2V_{,\varphi}} = \frac{3H^3}{2\cos\theta V_{,\varphi}}. \quad (25)$$

This is this number that is constrained by the observations: \mathcal{P}_0 is of the order of 2×10^{-4} (see for instance [27].)

One can be a bit more precise. The first term is expected to be a random field of power spectrum of index,

$$n_1 = 1 - 6\epsilon + 2\eta, \quad (26)$$

and the second of power spectrum index,

$$n_2 = 1 - 2\epsilon + 2\eta_1, \quad (27)$$

where ϵ is the same in the two cases (it is due to the variation of H),

$$\epsilon = -\frac{\dot{H}}{H^2}, \quad (28)$$

and η are the masses of the fields in units of H in respectively the adiabatic direction and the transverse direction,

$$\eta = \frac{M_{\text{Pl}}^2}{V} \frac{\partial^2 V}{\partial \varphi^2} \quad (29)$$

$$\eta_1 = \frac{M_{\text{Pl}}^2}{V} \frac{\partial^2 V}{\partial \chi_1^2}. \quad (30)$$

The resulting index, in first order in slow-roll parameter is then,

$$n = 1 + \cos^2 \theta (2\eta_1 - 6\epsilon) + \sin^2 \theta (2\eta_2 - 2\epsilon). \quad (31)$$

These parameters however vanish as soon as g is small and we are left with a scale free Harrison-Zel'dovich spectrum in all cases.

B. The induced non-Gaussianities

The most interesting consequences of such a family of models comes from the fact that they can potentially induce significant non-Gaussianities. By significant we mean that such non-Gaussianities can be much larger than what generic single field inflation predicts. The amount of non-Gaussianities which is generated is actually a direct consequence of the formula (22). Indeed, the self couplings of the field χ can be large. We are typically in a situation described in [12]. Those intrinsic non-Gaussianities are not due to any coupling of the field to the metric but to the self-coupling this scalar degree of freedom in a quasi-de Sitter background.

In particular, it is clear that $\delta\mathcal{S}_2$ can develop significant high order correlation function, its four point can be computed explicitly. Finite volume effects though can induce as well non-vanishing three-point function. As shown in [12] it amounts to shift the minimum of the field to a non zero value. The effective potential in the transverse directions is actually expected to be

$$V(\chi_1, \chi_2) = \frac{\nu^2}{4} [(\chi_1 + \bar{\chi}_1)^2 + (\chi_2 + \bar{\chi}_2)^2]^2, \quad (32)$$

where $\bar{\chi}_1$ and $\bar{\chi}_2$ are in essence random variables but that can be considered as fixed for our observable universe. A classical stochastic approach can be applied to infer the probability distribution function of its value. The Fokker equation for the joint distribution of $\bar{\chi}_1$ and $\bar{\chi}_2$ can be derived from the joint evolution of $\bar{\chi}_1$ and $\bar{\chi}_2$. It is given by,

$$\frac{\partial \mathcal{P}}{\partial t} = \frac{H^3}{8\pi^2} \left(\frac{\partial^2 \mathcal{P}}{\partial \bar{\chi}_1^2} + \frac{\partial^2 \mathcal{P}}{\partial \bar{\chi}_2^2} \right) + \frac{1}{3H} \left[\frac{\partial}{\partial \bar{\chi}_1} \left(\frac{\partial V(\bar{\chi}_1, \bar{\chi}_2)}{\partial \bar{\chi}_1} \mathcal{P} \right) + \frac{\partial}{\partial \bar{\chi}_2} \left(\frac{\partial V(\bar{\chi}_1, \bar{\chi}_2)}{\partial \bar{\chi}_2} \mathcal{P} \right) \right], \quad (33)$$

where t is the physical time. The late time solution of this equation is

$$\mathcal{P}(\bar{\chi}_1, \bar{\chi}_2) = \frac{\sqrt{2}\nu}{H^2\sqrt{3\pi}} \exp \left[-\frac{2\pi^2\nu^2 (\bar{\chi}_1^2 + \bar{\chi}_2^2)^2}{3H^4} \right], \quad (34)$$

for the correctly normalized distribution function. For the bispectrum, we will see that the parameter of interest is $\bar{\chi}_1$. Its distribution function, once marginalized over

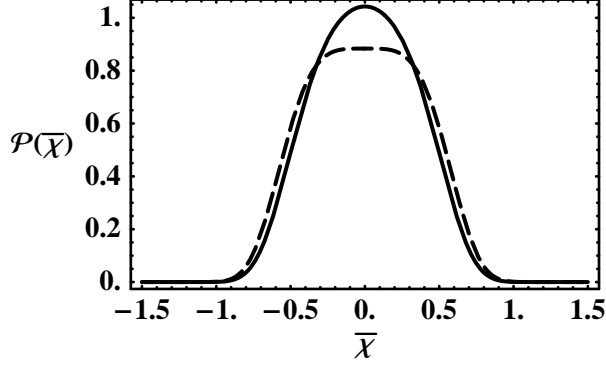


FIG. 1: Shape of the one-point distribution function of $\bar{\chi}_1$ for $H = 1$ and $\nu = 1$. The solid line corresponds to the equation (35) where the imaginary part of $\bar{\chi}$ has been integrated out. The dashed line corresponds to the case where $\bar{\chi}$ is real.

$\bar{\chi}_2$ is given by,

$$\mathcal{P}(\bar{\chi}_1) = \frac{2\nu|\bar{\chi}_1|}{H^2\sqrt{3\pi}} K_{\frac{1}{4}} \left(\frac{\pi^2\nu^2\bar{\chi}_1^4}{3H^4} \right) \exp \left[-\frac{\pi^2\nu^2\bar{\chi}_1^4}{3H^4} \right], \quad (35)$$

where $K_{\frac{1}{4}}$ is the modified Bessel function of the second kind of index 1/4. The shape of this PDF is shown on Fig. 1 for $H = 1$ and $\nu = 1$.

For generic initial conditions, and for a large enough number of e-folds prior to the horizon crossing of the modes of interests, $\bar{\chi}_1$ is expected to be distributed according to (35). The excursion domain for $\bar{\chi}_1$ is therefore typically $[-0.6H/\sqrt{\nu}, 0.6H/\sqrt{\nu}]$. As suggested before $\bar{\chi}_1$ can be seen as a free variable in a situation similar to that encountered in the curvaton scenario. Note that, from the potential (32), the field χ_1 acquires a mass equal to $6^{1/2}\nu\bar{\chi}_1$. It is generically small compared to H ensuring that the field can indeed develop super-Hubble fluctuations.

C. The perturbative regime

We are interested here in the high order (e.g. three- and four-point) correlation functions of the Fourier modes of the field χ and consequently δN . For a field $\sigma(\mathbf{x})$ we define its Fourier transform $\sigma(\mathbf{k})$ by,

$$\sigma(\mathbf{k}) = \int \frac{d^3\mathbf{x}}{(2\pi)^3} \sigma(\mathbf{x}) e^{-i\mathbf{k}\cdot\mathbf{x}}. \quad (36)$$

They read,

$$P_4^{\chi_1}(\mathbf{k}_1, \dots, \mathbf{k}_4) = 4\nu^4 N_e^2 \frac{9\bar{\chi}_1^2 + \bar{\chi}_2^2}{9H^4} [P(k_1)P(|\mathbf{k}_1 + \mathbf{k}_2|)P(k_3) + \text{sym.}] - 2\nu^2 N_e \frac{1}{H^2} [P^{\chi_1}(k_1)P^{\chi_1}(k_2)P^{\chi_1}(k_3) + \text{sym.}]. \quad (42)$$

This leads to

$$P_4^{\delta N}(\mathbf{k}_1, \dots, \mathbf{k}_4) = 4f_{\text{NL}}^2 \left(\frac{9\bar{\chi}_1^2 + \bar{\chi}_2^2}{9\bar{\chi}_1^2 \sin^2 \theta} \right) [P(k_1)P(|\mathbf{k}_1 + \mathbf{k}_2|)P(k_3) + \text{sym.}] + 6g_{\text{NL}} [P(k_1)P(k_2)P(k_3) + \text{sym.}]. \quad (43)$$

The power spectrum $P^\sigma(k)$ and the higher order spectrum $P_n^\sigma(k)$ are defined by

$$\begin{aligned} \langle \sigma(\mathbf{k}_1) \sigma(\mathbf{k}_2) \rangle &= \delta_{\text{Dirac}}(\mathbf{k}_1 + \mathbf{k}_2) P^\sigma(\mathbf{k}_1), \\ \langle \sigma(\mathbf{k}_1) \dots \sigma(\mathbf{k}_n) \rangle_c &= \delta_{\text{Dirac}}(\mathbf{k}_1 + \dots + \mathbf{k}_n) P_n^\sigma(\mathbf{k}_1, \dots, \mathbf{k}_n), \end{aligned} \quad (37)$$

where $_c$ denotes the connected parts.

A full quantum calculation can be done at tree order in the perturbative regime (see last section for details). In a de Sitter background the exact shape can be obtained for the three and connected part of the four-point function for the potential (32). In the super-Hubble regime they read [41],

$$P_3^{\chi_1}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = -\frac{2\nu^2 N_e \bar{\chi}_1}{H^2} [P^{\chi_1}(k_1)P^{\chi_1}(k_2) + \text{sym.}] \quad (39)$$

where N_e is the number of e-folds between horizon crossing and the end of inflation.

As stressed in previous papers, what drives the amplitude of fluctuations is the value of $\nu^2 N_e$ if $\bar{\mathcal{S}}$ is of the order of H . This result can be expressed phenomenologically through an f_{NL} parameter. It is defined in such a way that the bispectrum $P_3^{\delta N}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ of the metric fluctuation reads,

$$P_3^{\delta N}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = 2f_{\text{NL}} [P(k_1)P(k_2) + \text{sym.}], \quad (40)$$

when written in terms of the metric power spectrum. Relation (39) implies that,

$$f_{\text{NL}} = -\nu^2 N_e \frac{\bar{\chi}_1}{H} \frac{\sin^3 \theta}{2P_0}. \quad (41)$$

From the adopted definition of f_{NL} , the metric fluctuation will be significantly non-Gaussian if $P_0 f_{\text{NL}}$ approaches unity whereas standard inflationary physics implies that f_{NL} is of the order of unity [3].

Within the framework of this model, it is also possible to compute any higher order correlation functions. When $\bar{\chi}_1$ and $\bar{\chi}_2$ are both taken into account, there are two terms contributing (and from a field theory point of view, it means that there are three- as well as four-leg vertices that contribute).

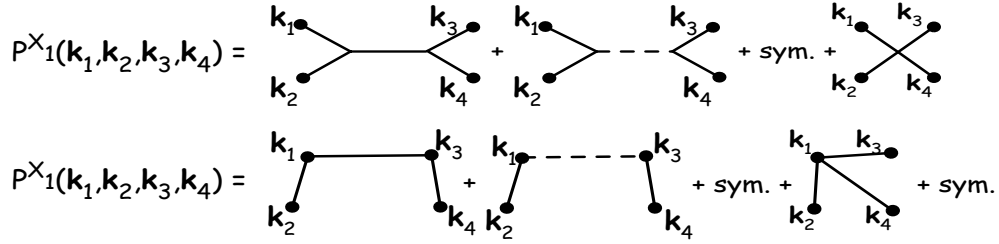


FIG. 2: Diagrammatic representation of the contributions to the connected four point functions of the χ_1 component of the complex field χ . The upper line show the Feynman type diagrams that have to be considered in a quantum field approach (for details see for instance [28]). For tree order calculations one can equivalently use a classical approach (provided the initial stochastic fields have the properties derived from the free field quantum solutions). The bottom line shows the resulting diagrams where each vertex point then represents a given order in the initial field. The mode dependence of those terms can then easily be derived in the super-horizon limit: the three diagrams that are represented give respectively $P^{\chi_1}(k_2)P^{\chi_1}(|\mathbf{k}_1 + \mathbf{k}_2|)P^{\chi_1}(k_4)$, $P^{\chi_1}(k_2)P^{\chi_2}(|\mathbf{k}_1 + \mathbf{k}_2|)P^{\chi_1}(k_4)$, $P^{\chi_1}(k_2)P^{\chi_1}(k_3)P^{\chi_1}(k_4)$.

The two terms of this equation correspond to two different geometries. The coefficient g_{NL} reads,

$$g_{\text{NL}} = -\nu^2 N_e \frac{\sin^4 \theta}{12\mathcal{P}_0^2}. \quad (44)$$

Note that the expression of the bispectrum and the second term of the tri-spectrum is what a nonlinear transform of the type,

$$\delta N = \delta N + f_{\text{NL}}(\delta N)^2 + g_{\text{NL}}(\delta N)^3 + \dots \quad (45)$$

would have given. Such a transform would however lead to $4f_{\text{NL}}^2$ for the coefficient of the first term of the tri-spectrum. The difference is due to the fact that the resulting metric fluctuations are actually the superposition of different fields, not only the nonlinear transform of a single one.

The relative weight of these two contributions depends in particular on the value of $\overline{\chi}_1$, and therefore on the peculiar realization of our own universe. These results are valid only in the perturbative regime, e.g. with respect to the coupling constant. It appears that the expansion parameter is different during sub-Hubble physics and during super-Hubble evolution. In the former case, ν^2 is the parameter to be small. When it is large the field fluctuations simply vanish away. At super-Hubble scale however one finds that the effective expansion parameter is actually $\nu^2 N_e$ which is obviously much larger than ν^2 . It limits the formal validity of the previous expressions. It also opens a new specific phenomenological domain the tentative description of which is the subject of the next section.

In Fig. 3, we compare the computed amplitude of the bi- and tri-spectrum to the current (with WMAP) and expected (with Planck) constraints. The left panel corresponds to a mixing angle $\theta = \pi/4$, that corresponds to an equal contribution of the two fields to the metric fluctuations, and the right panel to a small mixing angle,

$\theta = 0.1$. We restricted the parameter space to two variables, ν and $\overline{\chi}_1$, assuming $\overline{\chi}_2 = 0$ (it anyway contributed only weakly to the amplitude of the tri-spectrum). The dotted lines show the location of the expected generic value for $\overline{\chi}_1$. Values that would differ too much (in logarithmic space) from this location would demand some fine tuning in the peculiar random realization of the universe we live in.

In both cases the thick solid line shows the current constraint provided by the amplitude of the bispectrum. It naturally limits a combination of $\overline{\chi}_1$ and ν . When the mixing angle is large, left panel, it generically leads to small values of ν . The use of a perturbation theory is then fully justified. When the mixing angle is small the observations are much less efficient in constraining ν . It can be as large as unity. It is then necessary to reconsider the calculations that have been done to take into account non-perturbative aspects of the super-Hubble evolution of the fields.

The Planck mission is expected to provide us with much more stringent constraints not only on the bispectrum but also on the amplitude of the tri-spectrum. In Fig. 3 the dashed lines show the location of the bispectrum constraint provided by Planck. Regarding the tri-spectrum we have not attempted to take into account the different geometrical patterns that appear in its theoretical expression. As a result the amplitude of the trispectrum results of the summation of two terms of opposite signs. There is therefore a location in the parameter space where the tri-spectrum effectively vanishes. It is shown as a long dashed line. The constraint that observations would provide is given in terms of τ_{NL} which is set to be equal to $6g_{\text{NL}} + 4f_{\text{NL}}^2/\sin^2(\theta)$. The gray area is the region that Planck could be able to exclude according to [29]. It appears clearly that for low values of $\overline{\chi}_1$, the tri-spectrum is more efficient in constraining the amplitude of the coupling constant ν . In the rare event tails for $\overline{\chi}_1$, and along the cancelation line, the bispectrum

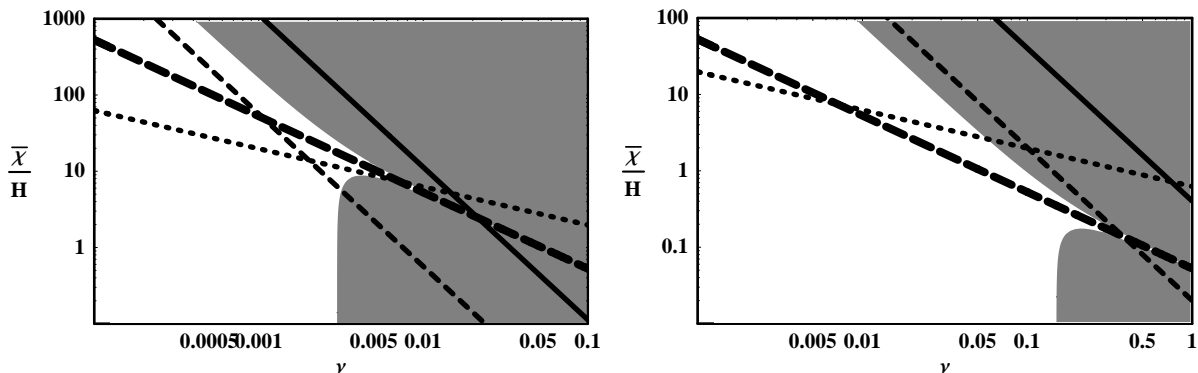


FIG. 3: Exclusion diagrams for parameters ν and $\bar{\chi}$ for $\theta = \pi/4$ (left panel) and for $\theta = 0.1$ (right panel). The locations of the dotted lines where $\bar{\chi}$ is equal to its expected one σ fluctuation. The gray areas and solid or short dashed lines correspond to the exclusion zones, obtained by WMAP (solid line) or expected by Planck (short dashed for bispectrum, gray areas for trispectrum). The bispectrum constraint corresponds to a straight line (of slope -2); the trispectrum is more complicated due to two competing terms in the trispectrum. The long dashed is the location where the terms cancel. We adopted the results of [29] on the upperbounds the Planck mission is expected to provide, $f_{\text{NL}} = 5$ and $\tau_{\text{NL}} = 560$.

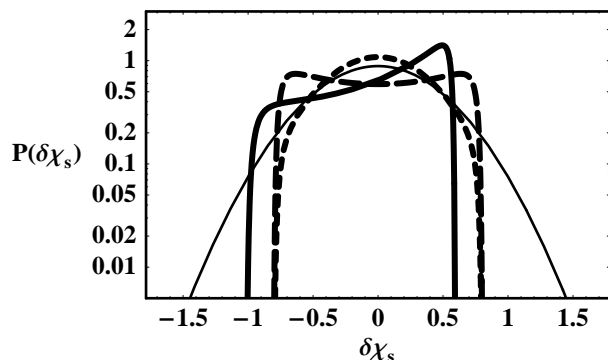


FIG. 4: Example of shapes of the one-point PDF of the local value of χ in case it underwent a non-perturbative evolution at super-horizon scales. The plots correspond to parameter values $\nu N_e^{1/2} = 1.5$; the long dashed line to $\bar{\chi} = 0$; the solid line to $\bar{\chi}_1 = 0.5$, $\bar{\chi}_2 = 0$ and the short dashed line to $\bar{\chi}_1 = 0$, $\bar{\chi}_2 = 2$ (values of $\bar{\chi}$ are given in units of H). The thin solid line is a Gaussian distribution of similar width. The resulting PDF would be the convolution of one of the first distribution with a Gaussian one with arbitrary relative amplitude.

is more efficient. The two observations appear therefore very complementary. For low values of θ though, it is obviously more difficult to get strong constraints of the coupling constant ν . In this case the field \mathcal{S}_2 could actually induce primordial NG when it enters a classical non-perturbative regime.

D. The non-perturbative regime

The aim of this section is to point to the existence of a regime where the statistical properties of the field \mathcal{S}_2 are transformed due to its super-Hubble nonlinear evolution. Such properties could be transferred in the observed met-

ric perturbation in case of a small mixing angle.

Let us recall that at super-Hubble scales the evolution equation for each component of the field is given by,

$$3H\dot{\chi}_i = -\nu^2(\chi_i + \bar{\chi}_i)|\chi + \bar{\chi}|^2 \quad (46)$$

for each component of the field and as long as the slow roll conditions are valid. An equation that obviously can be solved explicitly in the classical regime. It leads to,

$$\chi_i(t) = \frac{\chi_i^{\text{HC}} + \bar{\chi}_i}{\left[1 + 2\nu^2|\chi^{\text{HC}} + \bar{\chi}|^2 \int \frac{dt}{3H}\right]^{1/2}} - \bar{\chi}_i \quad (47)$$

where χ^{HC} is the value of χ at horizon crossing. It is important to note here that large NG will classically develop when $\nu^2 N_e$ is larger than unity whereas the self-coupling of χ at horizon crossing is driven by ν^2 . There is thus a domain in parameter space where the nonlinear evolution of the field fluctuations is essentially classical.

Using equation (47) and assuming that χ^{HC} is Gaussian distributed, one can compute explicit PDFs of χ_1 . They are presented on Fig. 4 for specific values of the $\bar{\chi}$ parameters. One observes that for significant values of the coupling constant, the large excursion values of χ_1 are strongly suppressed.

It is then tempting to assume that the resulting field properties can be derived by the application of such a local transform. That would indeed lead to interesting phenomenology regarding the local metric properties. It is however an approximation the importance of which is difficult to grasp. Indeed the relation (47) would be valid if the fluctuations had all the same scale and cross the horizon at the same time. This is not so. For instance when one observes fluctuation at 10 Mpc scale, its nonlinear evolution is bound to be affected by modes at much smaller scales, that have crossed the horizon later, but that nonetheless affect the nonlinear evolution of the field

by changing the local field values. In a field theory language, this is a UV effect in the radiative correction. Its importance cannot be neglected since one expects modes that are up to e^{60} smaller than the observed ones to cross the horizon during the super-Hubble evolution of the observed modes.

The detailed description of these effects is left for a forthcoming study. The transform (47) however suggests that the main effect of such a fully nonlinear evolution of the field is to squeeze the rare event tails. Such an effect would be visible from the behavior of the moments of the distributions shown on Fig. 4.

IV. CONCLUSIONS

We show here that it is possible to build models of multiple field inflation in a supersymmetric context that can generate a significant amount of non-Gaussianity. The models we advocate here are natural extension of D-term inflationary models. As models producing primordial non-Gaussianities, these types of models fall in the same class as those described in [13]. The only difference is that the initial isocurvature fluctuations are produced by a complex scalar field (not a real) which (mildly) affects its high-order correlation properties.

Such models are characterized by a mixing angle and an intrinsic self-coupling parameter. In SUSY context, when one wants to avoid fine tuned terms in the superpotential, the potential is naturally quartic and such models are remarkably constrained. In particular the shape of the inflationary potential has only a limited number of parameters. We note that in such a setting the mass of the isocurvature modes is automatically protected against radiative corrections (as an application of the results presented in [30]). In our analysis we have focused on a regime suggested by the observational constraints regarding cosmic strings formations. Those constraints impose that the Hubble scale is essentially constant during the inflationary phase which makes the transfer of modes, from isocurvature to adiabatic, potentially very efficient. Finite volume effects also introduce new dynamically determined parameters that induce effective cubic terms in the potential [31] the consequence of which we also present.

The phenomenological consequences of such a family of models can be obtained analytically. The induced high order correlation functions are due to super-Hubble evolution of the field. This opens the possibility of having weakly non-Gaussian fields. Two parameter domains can be distinguished for its phenomenology. If the mixing angle is generic then the current constraints suggest that the coupling parameter is small enough so that a tree order calculation of a perturbation theory suffice to derive the bi- and tri-spectra of the metric field. On the other hand if the mixing is small enough (0.1 for instance), then the resulting metric statistical properties might have been shaped by a nonlinear evolution of the field during

its super-Hubble evolution. In principle, such an evolution can nonetheless be addressed analytically because, at such scales, the fields enter a purely classical behavior. We present some tentative description of the evolution of the high order moments of the field.

The set of predictions we have obtained has eventually been confronted to the current constraints and to the expected constraints that the Planck satellite is expected to provide us for. It is shown that a joint use of CMB temperature bispectrum and trispectrum is required to efficiently explore the parameter space of such models. This, with the curvaton mechanism, is one of the very few explicit models in which primordial NG can significantly exceeds those naturally induced by the gravitational dynamics. In this case, it is fully determined by the inflationary sector of the theory.

V. CORRELATORS OF A TEST FIELD IN DE SITTER SPACE

We review here the formal expression of the connected bi- and tri-spectrum of the component of a test scalar complex field in a de Sitter background. We assume that the potential contains cubic and quartic terms,

$$V(\chi) = \nu^2 \left(\frac{1}{4} \chi_1^4 + \chi_1^3 \bar{\chi}_1 + \chi_1^2 \chi_2 \bar{\chi}_2 + \dots \right) \quad (48)$$

as implied by the form (32). The dots represent terms that won't affect the (tree order) expressions of the bi- and tri-spectrum of χ_1 .

The perturbative calculations of those correlators can be done with the In-In formalism (see [2, 28]). If Q is the quantity the vacuum expectation value one wants to compute then it can be shown that a perturbative expansion is given by

$$\langle 0 | Q(\chi(\eta)) | 0 \rangle = \int_{\eta_0}^{\eta} d\eta_1 \int_{\eta_0}^{\eta_1} d\eta_2 \dots \langle 0 | [\dots [Q(\chi^{(0)}(\eta)), -iH^{(I)}(\eta_1)], -iH^{(I)}(\eta_2)] \dots] | 0 \rangle, \quad (49)$$

where $H^{(I)}(\eta)$ is the interaction part of the Hamiltonien, η_0 is an arbitrarily early time when the interaction is supposed to start playing a role, η is the time at which the expectation value is computed and $\chi^{(0)}(\eta)$ are the field values at time η when they evolve according to the non-interacting part of the Lagrangian. In the following we use this formulae for $Q = \chi_1(\mathbf{k}_1) \dots \chi_1(\mathbf{k}_3)$, $Q = \chi_1(\mathbf{k}_1) \dots \chi_1(\mathbf{k}_4)$ and for $H^{(I)} = \int d^3\mathbf{x} \sqrt{-g} V(\chi)$ while keeping the expansion only to tree order.

The expression of those correlators will depend on the time dependance of the free field. The latter is given by

$$\chi_i^{(0)}(\mathbf{x}) = \frac{1}{a} \int d^3\mathbf{k} \left(f_k(\eta) a_{\mathbf{k}} e^{i\mathbf{k}\mathbf{x}} + f_k^*(\eta) a_{\mathbf{k}}^\dagger e^{-i\mathbf{k}\mathbf{x}} \right) \quad (50)$$

where,

$$f_k = \frac{1}{\sqrt{2k}} \left(1 + \frac{i}{k\eta} \right) e^{ik\eta} \quad (51)$$

for a de Sitter background. η is here the conformal time, $\eta = -1/(aH)$. this function permits to define the different time Green function, $G_k(\eta, \eta')$,

$$G_k(\eta, \eta') = f_k(\eta') f_k^*(\eta). \quad (52)$$

After some calculations, one can show that the tree order term for the three-point function is given by,

$$P_3^{\chi_1}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = 6i\nu^2 \bar{\chi}_1 \frac{1}{a^3} \int_{-\infty}^{\eta} \frac{d\eta'}{H\eta'} [G_{k_1}(\eta, \eta') G_{k_2}(\eta, \eta') G_{k_3}(\eta, \eta') - \text{c.c.}] \quad (53)$$

It is interesting to define a reduced bispectrum $Q_3(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ as

$$Q_3(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = \frac{P_3^{\chi_1}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)}{P^{\chi_1}(k_1) P^{\chi_1}(k_2) + \text{sym.}}. \quad (54)$$

This function is shown on Fig. 5 as a function of η , or more precisely as a function of the number of efolds since (or before) horizon crossing. The mode configuration correspond to an equilateral configuration ($k_1 =$

$k_2 = k_3$) but the asymptotic behavior of Q_3 is independent on the configuration (dashed line) if N_e is defined as $N_e = -\log(-k_t \eta)$ with $k_t = k_1 + k_2 + k_3$.

For the four point function two types of diagrams are contributing at tree order. One is due to the quartic part of the potential. In the following we will denote it as the "star" contribution. The other is due to the cubic terms of the potential. There are two such contributions (one due to an exchange of χ_1 degree of freedom and one to an exchange of χ_2). Although a priori smaller than the previous term, these two contributions are not necessarily negligible.

The star contribution is given by [32],

$$P_{\text{star}}^{\chi_1}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) = -6i\nu^2 \frac{1}{a^4} \int_{-\infty}^{\eta} d\eta' [G_{k_1}(\eta, \eta') G_{k_2}(\eta, \eta') G_{k_3}(\eta, \eta') G_{k_4}(\eta, \eta') - \text{c.c.}] \quad (55)$$

The line contribution is finally given by the formal expression,

$$P_{\text{line}}^{\chi_1}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) = -2\nu^4 (9\bar{\chi}_1^2 + \bar{\chi}_2^2) \frac{1}{a^4} \int_{-\infty}^{\eta} \frac{d\eta_1}{H\eta_1} \frac{d\eta_2}{H\eta_2} [G_{|\mathbf{k}_1+\mathbf{k}_2|}(\eta_1, \eta_2) + \text{c.c.}] [G_{k_1}(\eta, \eta_1) G_{k_2}(\eta, \eta_1) - \text{c.c.}] [G_{k_3}(\eta, \eta_1) G_{k_4}(\eta, \eta_1) - \text{c.c.}] + \text{sym.} \quad (56)$$

Similarly to Q_3 one can define the reduced tri-spectra as[42],

$$Q_{\text{star}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) = \frac{P_{\text{star}}^{\chi_1}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4)}{P^{\chi_1}(k_1) P^{\chi_1}(k_2) P^{\chi_1}(k_3) + \text{sym.}}. \quad (57)$$

and

$$Q_{\text{line}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) = \frac{P_{\text{line}}^{\chi_1}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4)}{P^{\chi_1}(k_1) P^{\chi_1}(k_3) P^{\chi_1}(|\mathbf{k}_1 + \mathbf{k}_2|) + \text{sym.}}. \quad (58)$$

This function is shown on right panel of Fig. 5 for a peculiar configuration. Note that the relative importance of the two depends on the configuration even in the asymptotic limits.

The asymptotic limit of the line part of the tri-spectrum is given by,

$$P_4^{\text{line}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) \rightarrow -\frac{\nu^4 (9\bar{\chi}_1^2 + \bar{\chi}_2^2)}{72 H^4 k^3 k_1^3 k_2^3 k_3^3 k_4^3} [q_4(\mathbf{k}_1, \mathbf{k}_2, \eta) q_4(\mathbf{k}_3, \mathbf{k}_4, \eta) + \text{sym.}] \quad (59)$$

with

$$q_4(\mathbf{k}_1, \mathbf{k}_4, \eta) = 2(k_1 + k_2)k^2 + (k_1^3 + k_2^3)(2 - 2\gamma) + 2k_1 k_2 (k_1^2 + k_2^2) + k^3 \log[(k_1 + k_2 + k)/(k_1 + k_2 - k)] - (k_1^3 + k_2^3) \log[\eta^2 (k_1 + k_2 + k)(k_1 + k_2 - k)], \quad (60)$$

where $k = |\mathbf{k}_1 + \mathbf{k}_2|$ and γ is the Euler number. Note that

the dominant contribution of q_4 at super-horizon scale

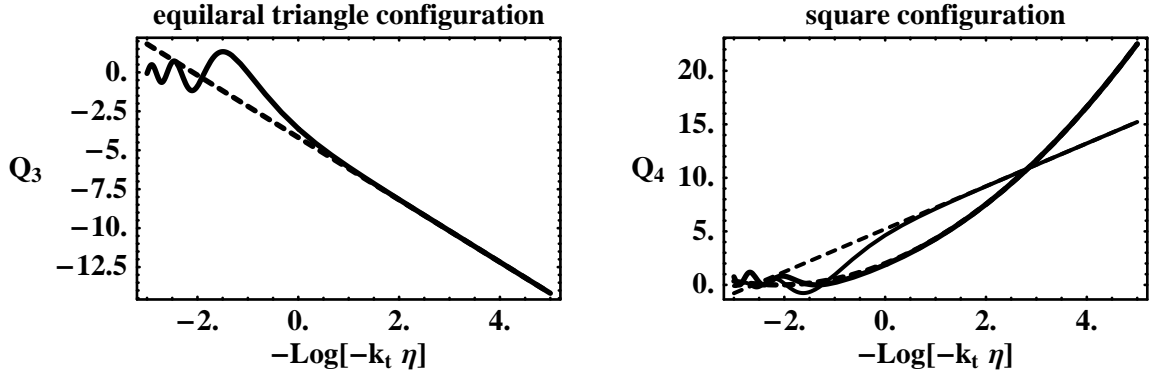


FIG. 5: Behavior of the reduced correlators, Q_3 and Q_4 of the field χ_1 , as a function of $N_e \equiv \log(k_t \eta)$. The function Q_3 (left panel) is to be multiplied by $\nu^2 \bar{\chi}_1 / H^2$; the function Q_4^{star} (thick lines of right panel) is to be multiplied by $-\nu^2 / H^2$ and the function Q_4^{line} (thin lines) is to be multiplied by $2\nu^4 (9\bar{\chi}_1^2 + \bar{\chi}_2^2) / H^4$. The dashed lines correspond to the corresponding asymptotic behaviors.

is due to the last term. It behaves like $-2(k_1^3 + k_2^3)N_e$. This leads to the super-horizon behavior of the four point function of χ_1 exploited in the text.

Acknowledgments

FB thanks Galileo Galilei Institute, Florence, for its hospitality in the course of this work. The authors also

thank Jean-Philippe Uzan for fruitful discussions and comments regarding this work. FB is also partially supported by the the French Programme National de Cosmologie.

-
- [1] D. Seery and J. E. Lidsey, Journal of Cosmology and Astro-Particle Physics **6**, 3 (2005), astro-ph/0503692.
 - [2] J. Maldacena, Journal of High Energy Physics **5**, 13 (2003), astro-ph/0210603.
 - [3] N. Bartolo, E. Komatsu, S. Matarrese, and A. Riotto, Physics. Rep. **402**, 103 (2004), astro-ph/0406398.
 - [4] D. Seery and J. E. Lidsey, Journal of Cosmology and Astro-Particle Physics **1**, 8 (2007), astro-ph/0611034.
 - [5] C. T. Byrnes, M. Sasaki, and D. Wands, Phys. Rev. D **74**, 123519 (2006), astro-ph/0611075.
 - [6] N. Bartolo, S. Matarrese, and A. Riotto, Journal of Cosmology and Astro-Particle Physics **6**, 24 (2006), astro-ph/0604416.
 - [7] N. Bartolo, S. Matarrese, and A. Riotto, Journal of Cosmology and Astro-Particle Physics **1**, 19 (2007), astro-ph/0610110.
 - [8] E. Silverstein and D. Tong, Phys. Rev. D **70**, 103505 (2004), hep-th/0310221.
 - [9] M. Alishahiha, E. Silverstein, and D. Tong, Phys. Rev. D **70**, 123505 (2004), hep-th/0404084.
 - [10] D. H. Lyth and D. Wands, Physics Letters B **524**, 5 (2002), hep-ph/0110002.
 - [11] N. Bartolo, S. Matarrese, and A. Riotto, Phys. Rev. D **65**, 103505 (2002), hep-ph/0112261.
 - [12] F. Bernardeau and J.-P. Uzan, Phys. Rev. D **66**, 103506 (2002), hep-ph/0207295.
 - [13] F. Bernardeau and J.-P. Uzan, Phys. Rev. D **67**, 121301 (2003), astro-ph/0209330.
 - [14] A. Linde, Phys. Rev. D **49**, 748 (1994), astro-ph/9307002.
 - [15] A. Linde and P. Murdin, Encyclopedia of Astronomy and Astrophysics (2000).
 - [16] G. Dvali, Q. Shafi, and R. Schaefer, Physical Review Letters **73**, 1886 (1994), hep-ph/9406319.
 - [17] P. Binétruy and G. Dvali, Physics Letters B **388**, 241 (1996), hep-ph/9606342.
 - [18] D. H. D. H. Lyth and A. A. Riotto, Physics. Rep. **314**, 1 (1999), hep-ph/9807278.
 - [19] S. Coleman and E. Weinberg, Phys. Rev. D **7**, 1888 (1973).
 - [20] R. Jeannerot and M. Postma, Journal of Cosmology and Astro-Particle Physics **7**, 12 (2006), hep-th/0604216.
 - [21] J. Rocher and M. Sakellariadou, Physical Review Letters **94**, 011303 (2005), arXiv:hep-ph/0412143.
 - [22] G. Felder, J. García-Bellido, P. B. Greene, L. Kofman, A. Linde, and I. Tkachev, Physical Review Letters **87**, 011601 (2001), hep-ph/0012142.
 - [23] G. Felder, L. Kofman, and A. Linde, Phys. Rev. D **64**, 123517 (2001), hep-th/0106179.
 - [24] A. A. Starobinskii, Soviet Journal of Experimental and Theoretical Physics Letters **42**, 152 (1985).
 - [25] M. Sasaki and E. D. Stewart, Progress of Theoretical Physics **95**, 71 (1996), astro-ph/9507001.
 - [26] M. Sasaki and T. Tanaka, Progress of Theoretical Physics **99**, 763 (1998), gr-qc/9801017.
 - [27] A. R. Liddle and D. H. Lyth, *Cosmological Inflation and*

- Large-Scale Structure* (Cosmological Inflation and Large-Scale Structure, by Andrew R. Liddle and David H. Lyth, pp. 414. ISBN 052166022X. Cambridge, UK: Cambridge University Press, April 2000., 2000).
- [28] S. Weinberg, Phys. Rev. D **72**, 043514 (2005), hep-th/0506236.
 - [29] N. Kogo and E. Komatsu, Phys. Rev. D **73**, 083007 (2006), astro-ph/0602099.
 - [30] T. Brunier, F. Bernardeau, and J.-P. Uzan, Phys. Rev. D **71**, 063529 (2005), hep-th/0412186.
 - [31] F. Bernardeau and J.-P. Uzan, Phys. Rev. D **70**, 043533 (2004), astro-ph/0311421.
 - [32] F. Bernardeau, T. Brunier, and J.-P. Uzan, Phys. Rev. D **69**, 063520 (2004), astro-ph/0311422.
 - [33] D. N. Spergel, R. Bean, O. Dore, M. R. Nolta, C. L. Bennett, G. Hinshaw, N. Jarosik, E. Komatsu, L. Page, H. V. Peiris, et al., ArXiv Astrophysics e-prints (2006), astro-ph/0603449.
 - [34] F. Bernardeau, L. Kofman, and J.-P. Uzan, Phys. Rev. D **70**, 083004 (2004), astro-ph/0403315.
 - [35] T. Falk, R. Rangarajan, and M. Srednicki, Astrophys. J. Letter **403**, L1 (1993), astro-ph/9208001.
 - [36] The recent measurement of WMAP [33] on the primordial index is still a matter of debate.
 - [37] From this point of view such a model resembles modulated inflation models, in particular the one presented in [34].
 - [38] Inflation can actually end earlier
 - [39] Supergravity corrections can obviously be included in the expression of the superpotential [20]. The whole validity of a low energy effective theory approach remains however somewhat questionable in the absence of a full theory of quantum gravity if the VEVs of some fields approaches Planck scale.
 - [40] The amplitude of the power spectrum of the field fluctuations is $H^2/(2k^3)$.
 - [41] The original calculation is to be found in [35].
 - [42] There is here an ambiguity on how Q_4 can be defined since the two contributions do not have the same late time super-horizon behavior.